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Quantum Geometry from Coordinate Transformations Relating Quantum Observers

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The relativity principle that the law of propagation for light has the same form for all macroscopic observers is extended to include quantum observers; i.e., observers who may be large, but not infinitely large, by comparison with quantum mechanical systems. This leads to the extension of the covariance group from the diffeomorphisms to the conservation group (which is the largest group of coordinate transformations under which conservation laws are covariant statements) and, thus, to the quantum geometry and quantum unified field theory considered in a previous paper.

1. INTRODUCTION

In a previous paper (Pandres, 1981), hereafter referred to as I, we obtained a quantum unified field theory by pursuing a suggestion (Einstein, 1949) that the diffeomorphisms somehow be extended to a larger group. A diffeomorphism from coordinates x^α to x^i satisfies the commutation condition $[\partial_\mu, \partial_\nu]x^i = 0$. Our notation is that of I. In I, we replaced this commutation condition with the weaker condition

$$x^\nu{}_{,i}\left[\partial_\mu, \partial_\nu\right]x^i = 0 \tag{1}$$

and, showed that equation (1) defines a group which contains the diffeomorphisms as a proper subgroup. A conservation law is an expression of the form $\mathscr{V}^\alpha{}_{,\alpha} = 0$, where $\mathscr{V}^\alpha$ is a vector density of weight $+1$. We showed, in I, that equation (1) defines the largest group of coordinate transformations under which conservation laws are covariant statements. For this reason, we have called a coordinate transformation *conservative* if it satisfies equation

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(1), and, have called the group of all such transformations the *conservation group*. This group is so large that points do not have invariant meaning. (A conservative coordinate transformation from x^α to x^i is generally both one-to-many and many-to-one; hence, nonunique in both directions, even in finite coordinate patches.) Therefore, the geometry which is determined by the conservation group (as Riemann geometry is determined by the diffeomorphisms) is not defined on a differentiable manifold. It is defined on a space in which *paths* (which do have invariant meaning) are the primary elements. Path-space possesses properties which are sufficiently close to what one conceives of intuitively as a space so that one may use it almost exactly as one conventionally uses a differentiable manifold. Many investigators have expressed skepticism that a differentiable manifold adequately describes physical space-time. Because the primary elements are paths, it is natural to quantize the path-space geometry by using the path-integral method. By considering the macroscopic limit of the quantum geometry, in I, we obtained field equations which describe gravitation and electromagnetism, and which also contain terms that appear suitable for describing the weak and strong interactions. While these results are encouraging, it must be admitted that our introduction of the conservation group is lacking (thus far, at least) in the sort of compelling physical motivation which caused Einstein to introduce the diffeomorphisms. After all, there do exist observers who are accelerated with respect to one another, and, evidence of their equivalence is provided by the proportionality of inertial and gravitational mass. But, is there a need for the inclusion of some still more general class of observers?

2. QUANTUM OBSERVERS

General relativity makes use of a classical observer who can observe the motion of a physical system without disturbing the system. This violates the fundamental principles of quantum theory. Most discussions of observation in quantum theory make use of a "macroscopic" classical observer—one who can "stand outside" a quantum mechanical system and act upon the system without being acted upon by the system. This is unsatisfactory, because there exist no observers who are infinitely large by comparison with quantum mechanical systems. One solution of this problem was given by Everett (1957). He considers a quantum observer's memory to have quantum states that are correlated with the states of what he has observed. Each observer can then consider himself a macroscopic observer (since his different states are independent) and still treat other observers as part of his quantum mechanical universe. Each observer can also assign coordinates to events in his universe. The uncertainty principle does not limit the precision with which he can do this, because the four operators which represent his coordinates commute. Let Ω and O be two observers, each of whom considers himself a macroscopic observer, while treating the other as part of a quantum mechanical universe. Let x^α and x^i be coordinates which are assigned by Ω and O, respectively, to events in their universes. The assumption that physical space-time is a differentiable manifold rests squarely upon the assertion that it is possible to establish a one-to-one correspondence between x^α and x^i, at least in finite coordinate patches. Einstein challenged the validity of Newton's absolute time on the ground that no operational method had been or could be, given for its measurement. In this spirit, we challenge the validity of the assumption that space-time is a differentiable manifold on the ground that no operational method has been, or can be, given for establishing a one-to-one correspondence between the coordinates x^α and x^i. Our two observers are free to exchange information, so that, for example, Ω can possess a complete description of the procedure which O uses in assigning coordinates to events. If Ω could also state with certainty (as in general relativity) that O's world line is a particular path P, then he could write a function uniquely specifying O's coordinates x^i in terms of his own coordinates x^α. Thus x^i could be regarded as a functional of the path P (which has the terminus x^α since the observer in general relativity is a tetrad at the event being investigated)

$$x^i = x^i\{P\} \tag{2}$$

However, Ω has described O's world line as completely as nature permits when he states that *all* paths occur with equal probability amplitude, in the sense that the probability amplitude for a path P is $Ne^{iL\{P\}/\hbar}$, where L is Ω's Lagrangian for O, N is a normalization factor (the same for all paths), and $\hbar$ is the usual quantum of angular momentum. Therefore, Ω can only state that the probability amplitude for O's coordinate number x^i corresponding to his coordinate number x^α is

$$\Psi(x^i, x^\alpha) = \sum_P Ne^{iL\{P\}/\hbar} \tag{3}$$

where $\sum_P$ denotes the democratic sum with equal weight of contributions due to all paths with terminus x^α such that equation (2) yields the value x^i. As a world-point mapping, $\Psi(x^i, x^\alpha)$ is both one-to-many and many-to-one; hence, nonunique in both directions, as are our conservative coordinate transformations. As the size of observer O increases without limit, we find that the competing alternatives in equation (3) interfere destructively on all but the classically allowed path. Thus, equation (2) goes over to $x^i = x^i(x^\alpha)$ in the macroscopic limit. This just means that the group of all quantum coordinate transformations, defined by equations (2) and (3) contains the diffeomorphisms as a proper subgroup, as does our conservation group.

We are now in a position which permits us to present additional evidence that the inclusion of quantum observers, on an equivalent basis, requires the extension of the covariance group to the conservation group. We could simply say: "Experience shows that if Ω observes that a certain quantity is conserved, then O also observes that the same quantity is conserved." On the other hand, it is absolutely essential that such a fundamental principle as the covariance law be derivable from the simplest possible basic assumption. We therefore return to the assumption which led Einstein to special relativity—that the equation which describes the propagation of light (the wave equation) has the same form for all observers. It is well known that the wave equation may be written in the form $((-g)^{1/2} g^{\mu\nu} \Phi_{,\nu})_{,\mu} = 0$ in general relativity; and, that this form, which does not involve "covariant derivatives" or Christoffel symbols, is nevertheless covariant under the diffeomorphisms. We note that this general relativistic statement of the wave equation is already in the form of a conservation law: $\mathscr{V}^\mu{}_{,\mu} = 0$, where $\mathscr{V}^\mu$ is the vector density of weight $+1$ defined by $\mathscr{V}^\mu = (-g)^{1/2} g^{\mu\nu} \Phi_{,\nu}$. The results described immediately following equation (1) now suffice to show that the conservation group is the largest group of coordinate transformations under which the equation for the propagation of light is covariant.

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